A Tentative World Datum from Geoidal Heights Based on the Hough Ellipsoid and the Columbus Geoid

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Abstract—From all astro-geodetic material available at present, geoidal heights were computed on the 1927 North American Datum for the western hemisphere from Canada to Chile, and on the European Datum for the eastern hemisphere from Great Britain to Japan and from Scandinavia to South Africa. Improved reference ellipsoids were determined under various conditions for each hemisphere by the least squares method. Triaxiality is refuted. The Columbus Geoid, gravimetrically derived by W. A. Heiskanen, was used to connect the astro-geodetic systems of the two hemispheres. By minimizing the differences between the astro-geodetic and gravimetric geoid elevations at 75 points in the western hemisphere and 127 points in the eastern hemisphere, a theoretically absolute orientation based on the Hough ellipsoid was determined, leading to a tentative World Datum.

The use of geoidal heights rather than deflections of the vertical for a determination of the figure of the earth depends on the availability of geoidal contour maps for large contiguous areas. In Figure 1 the area covered is shown by hatching; the heavy lines indicate the long arcs of our preliminary study [*Chovitz* and *Fischer*, 1956], where the method of deflections of the vertical had been employed.

The western hemisphere—A geoidal contour map of North America on the 1927 North American Datum was completed recently [*Fischer*, 1957*a*]. Figure 2 is a generalization of this map, taking an initial value of zero at Meades Ranch instead of the 10 m for Calais, Maine, which had been used by *Fischer* [1957*a*] in conformity with *Hayford's* [1909] map. A geoidal profile was carried along the first-order triangulation arc through Central and South America with two eastward spurs, one through Bolivia into Brazil, the other through Colombia and Venezuela to Trinidad. In Trinidad this arc connects with the Hiran trilateration coming from Florida, closing the loop of about 10,000 km around the Caribbean Sea with a discrepancy of less than 25 m in position.

Figure 3 shows the geoidal profile on the 1927 North American Datum along the 100th meridian in North America and along the triangulation arc through Central into South America. The geoidal heights were computed according to the projection method, in which distances are reduced to the spheroid, not simply to the geoid as in the customary development method. *Molodenskiy's* [1944] technique was used to compute this additional correction (from geoid to spheroid), which is essentially a scale-factor correction. It is described in *Fischer* [1957*b*] in more detail. The effect on the geoidal profile is illustrated in Figure 3 as the difference between the full line (projection method) and the upper broken line (development method); it is negligible in North America, but accumulates to about 60 m at 40° South. The lower broken line is taken from the preliminary study [*Chovitz* and *Fischer*, 1956] where this arc was treated as a meridian, neglecting the η component of the deflection of the vertical.

The geoidal height relative to a reference ellipsoid depends on the size and shape of that ellipsoid and on its orientation with respect to the geoid, altogether on five parameters. By varying these five parameters and minimizing the geoidal heights in a least-squares solution, a best-fitting ellipsoid can be determined. The well known formula by de Graaff Hunter [*Survey of India*, 1939] was used for this purpose. To represent the western hemisphere, 131 observation equations were formed for points at 5° intervals. The origin was taken at Meades Ranch. The parameters of the best-fitting ellipsoid were determined as $f = 1/(297.1 \pm 0.2)$ and $a = 6,378,239 \pm 16\text{m}$ [*Fischer*, 1957*b*]. The geoid contours of North America referred to this ellipsoid are plotted in Figure 4. A profile along the 100th meridian, continuing into the Central

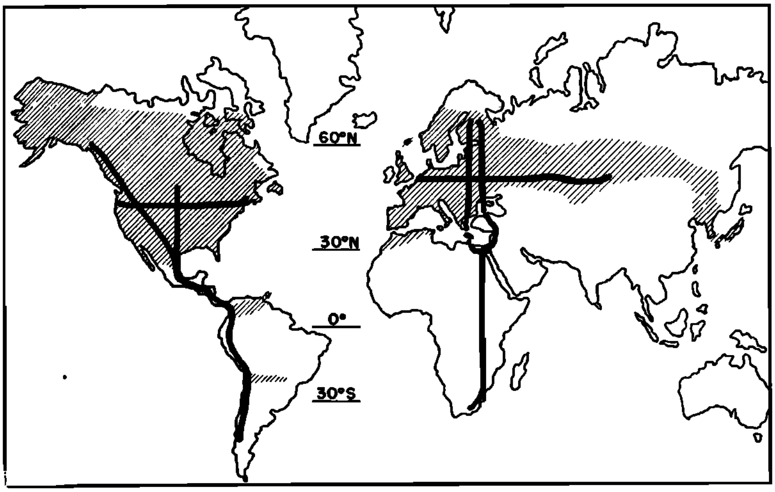

FIG. 1—Coverage

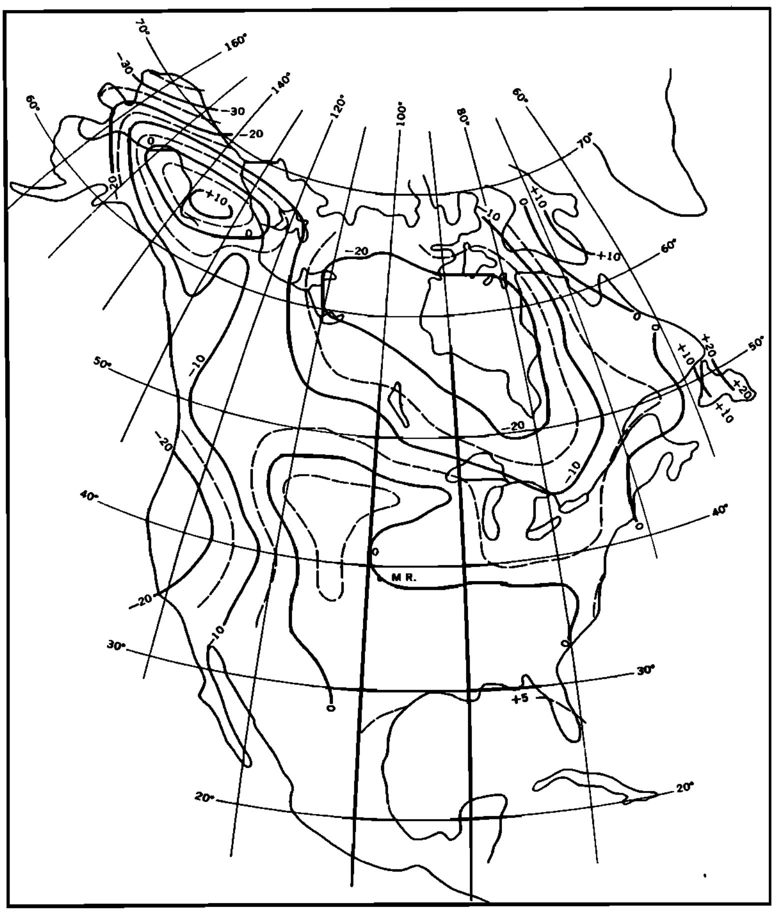

FIG. 2—North America, geoid contours in meters, 1927 North American Datum, assumed initial value zero meters at Meades Ranch (M.R.)

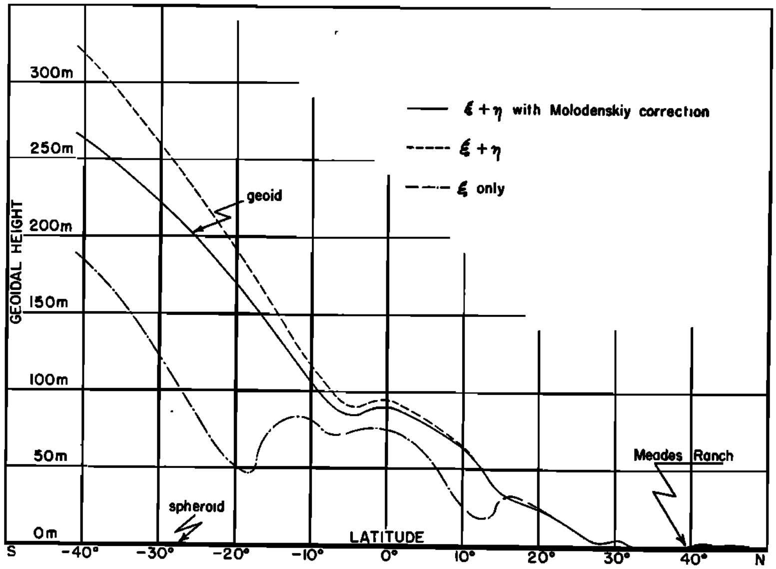

FIG. 3—Geoidal heights, America, 100th meridian, 1927 North American Datum

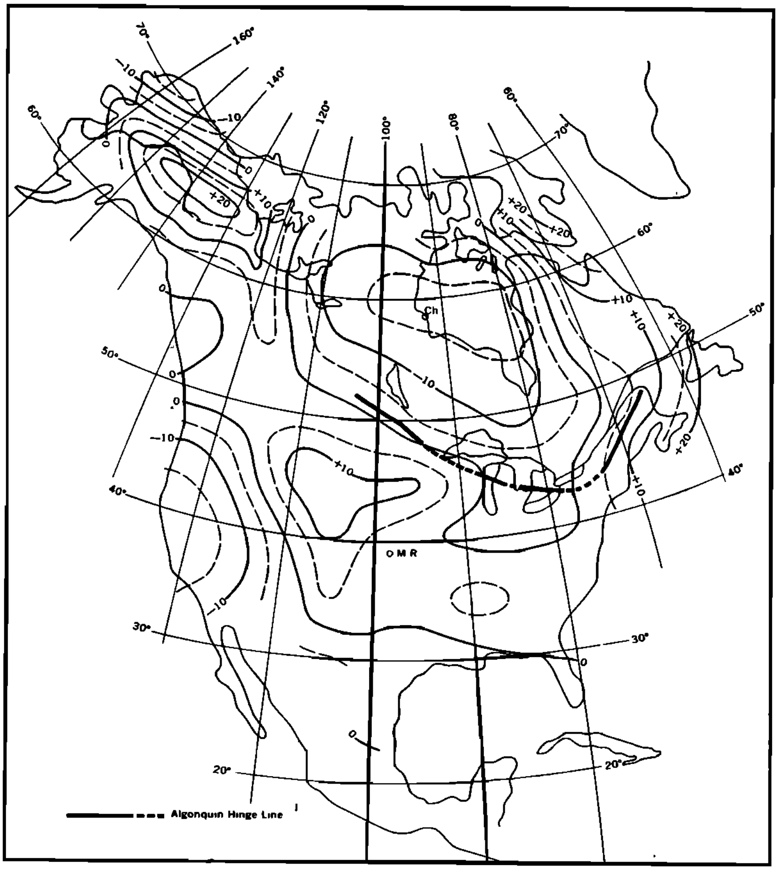

FIG. 4—North America, geoid contours in meters, best fitting ellipsoid; M.R. is Meades Ranch; Ch. is Churchill

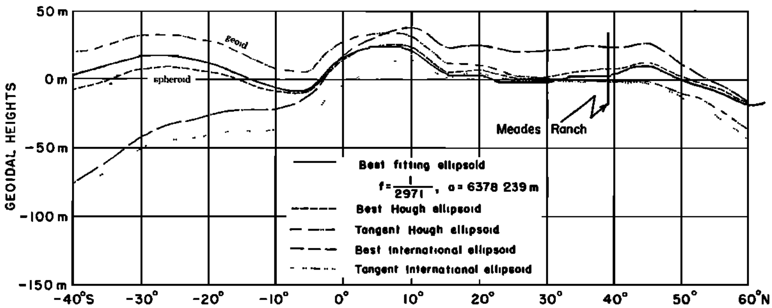

FIG. 5—Geoidal heights relative to various ellipsoids, America, 100th meridian

and South American arc, is plotted in Figure 5. This Figure also illustrates the difference in geoidal heights, if the best fitting ellipsoid were replaced by the International or the Hough ellipsoid, both in their best fitting orientation and in the case of tangency at the origin ($\delta\xi_0 = \delta\eta_0 = N_0 = 0$). The zero line represents each of the ellipsoids; the symbol of the curved line, representing the corresponding geoid profile, indicates to which ellipsoid the geoidal heights are referred. It can be seen that the International ellipsoid does not fit well outside the United States.

The Hough ellipsoid ($f = 1/297$, $a =$ 6,378,270 m) is a conventional surface used in problems of world-wide extent as in Project Vanguard.

The eastern hemisphere—The geoidal heights in Europe and North Africa on the European Datum were taken over from *Lieberman's* [1955] work. The addition of England, Scotland, and the Shetland Islands [*Assn. Internat. Geod.*, 1957] provided a loop closure to Norway. The connection from Greece via Crete to Africa changed the height of Crete to a value larger than that on Lieberman's chart; the change was confirmed by a loop closure in Africa through the use of an alternative route via the Levant. The Molodenskiy correction is insignificant for Europe but is quite large in South Africa. Its effect along the 30th meridian in Africa is shown in Figure 6, together with the geoidal profile along this arc.

The geoidal contour chart for the USSR was given on the Bessel ellipsoid of the 1932 triangulation [*Army Map Service*, 1953]. This reference ellipsoid and datum were changed to the International ellipsoid and European Datum by computing the separation between the two ellipsoids at stations covering the whole area at 5° intervals, making use of the known difference in position of Pulkovo. The underlying assumption here is that the Russian chart represents true geoidal heights according to the projection method. The Russian source gives no explanation on the method of computing these heights. The brief statement that they are based on triangulation data as of 1936 indicates that some correction should have been applied, since we know that at that time bases were not yet reduced to the ellipsoid as a general procedure. No mention of that correction having been made raised some doubt as to whether or not we should apply the Molodenskiy correction to the Russian material before converting it to the European Datum. In the paper presented at Toronto [*Fischer*, 1957*b*], both alternatives were pursued numerically. The amount of this correction is certainly not negligible, accumulating to about 25 m at the eastern end of the chart at 130°E, in case the original triangulation was computed on the Bessel ellipsoid. If a well fitting ellipsoid was used, the correction would be small. On the

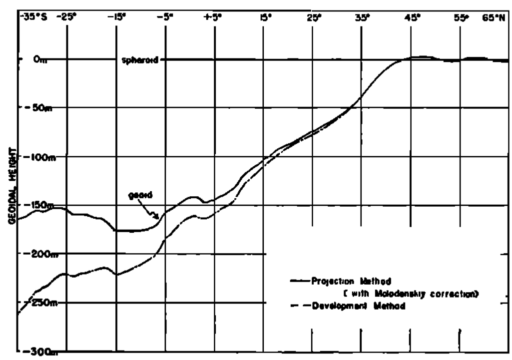

FIG. 6—Geoidal heights, Africa, European Datum, effect of Molodenskiy correction

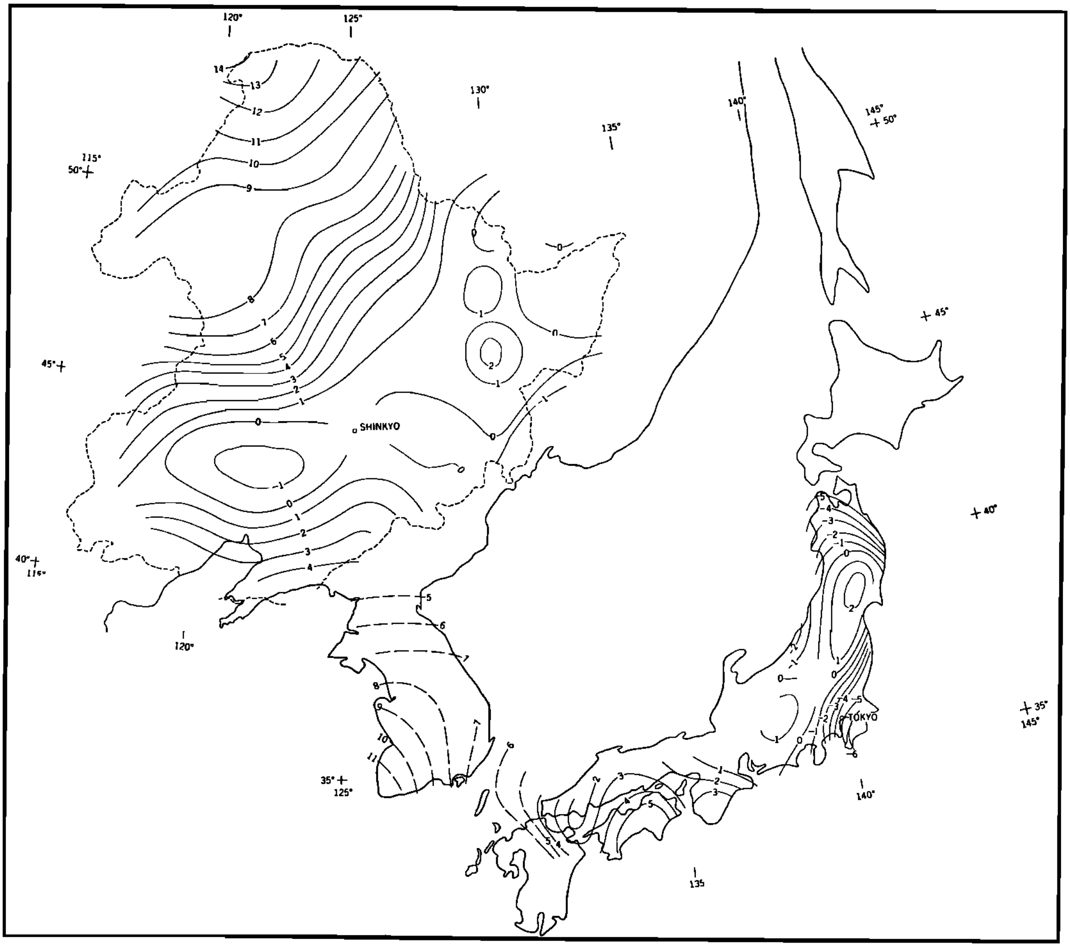

FIG. 7—Manchuria, Korea, Japan, geoidal contours in meters, Manchurian Principal System

other hand, we know from the historical background that at the time of Dubovskoy's publication in 1939 the distinction between development and projection methods was very well known and the implications had been discussed for more than a decade. Thus we feel justified in assuming for the present that these implications were considered when the chart was drawn. It is planned, however, to continue the alternative computations until the facts can be established.

The Manchurian material, given in the form of astro-geodetic deflections on the Bessel ellipsoid, was first used to compute geoidal contour lines on the Manchurian Datum by *Hayford*'s [1909] method; charts were drawn showing lines of equal deflections for the meridional and prime vertical components. From these charts geoidal heights were derived (Fig. 7) in the same way as for the North American map. No Molodenskiy correction seemed necessary here for the same reasons that held for Europe and North America. The Manchurian Datum could now be changed to the European Datum, since there is an overlap with the Russian net at 50°N, 130°E.

The astro-geodetic deflections in Japan on the Tokyo Datum [*Geogr. Surv. Inst.*, 1951, 1953, 1955; *Okuda*, 1951; *Torao*, 1951] were used in the same way to construct first lines of equal deflections, and from these, the geoidal contours of Figure 8. An extension of this chart was computed along a profile through Korea. A connection to Manchuria was computed, by which all of Japan and Korea could be converted into the Manchurian Principal System and from there into the European Datum. A com-

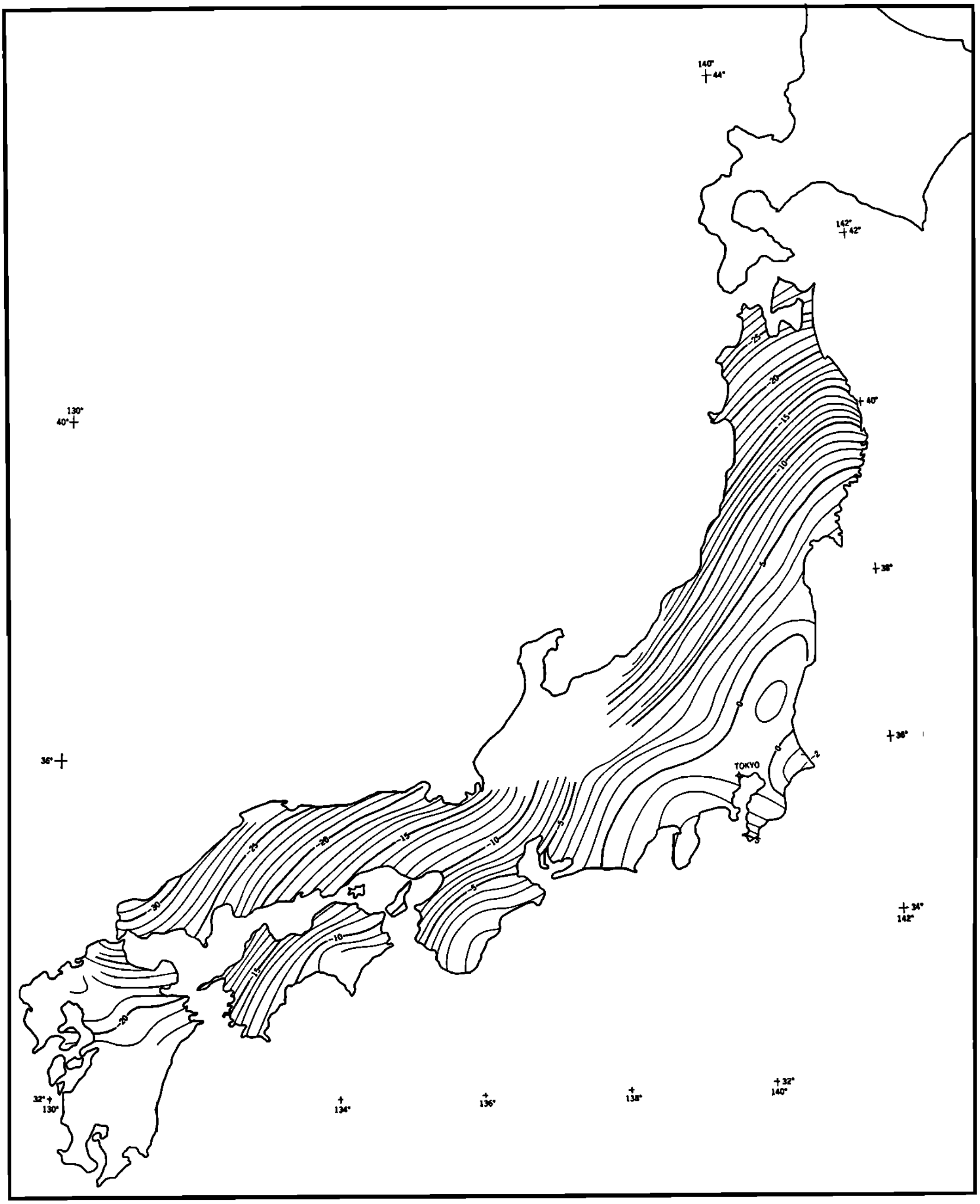

FIG. 8—Japan, geoidal contours in meters, Tokyo Datum

parison of the geoid contours of Japan on the Manchurian and Tokyo Datums (Figs. 7 and 8) reveals that the Manchurian Datum is much better suited to Japan than is its own datum. The reason is that Tokyo, the chosen origin, happens to be an atypical point, lying on a steep geoidal slope towards the Ramapo Deep in the Japanese trench. The resulting absolute deflection must be quite large. Since zero deflections at Tokyo are assumed as the basis for the Tokyo Datum, the geoidal heights of the country must appear as systematically negative. The deflection

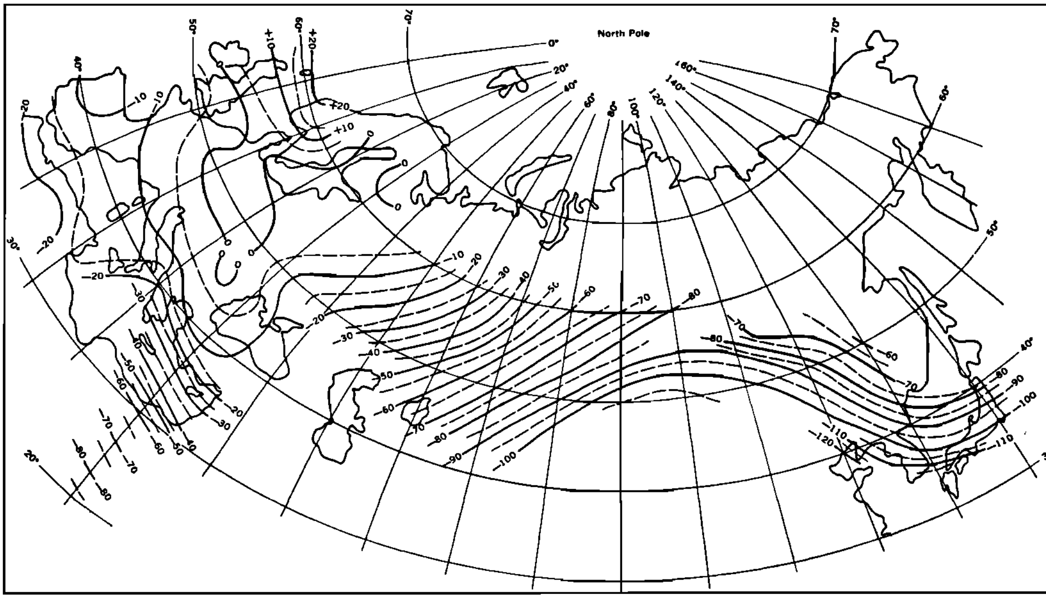

FIG. 9—Eastern hemisphere, geoid contours in meters, European Datum

of Tokyo on the Manchurian Principal System turns out to be of about the same order as, though somewhat larger than, the results found by *Atumi* [1933] and Kawabata [*Okuda,* 1951, p. 246] in their studies of a best-fitting ellipsoid.

The vast area from the Shetland Islands to Japan and from Scandinavia to South Africa is now unified on the European Datum (Figs. 6 and 9). The point of origin was chosen at 50°N, 25°E, and 137 equations were formed at 5° intervals. The parameters of the best-fitting ellipsoid are $f = 1/(297.1 \pm 0.2)$ and $a = 6{,}378{,}279 \pm 8$ m; these are somewhat larger than the figures given by *Fischer* [1957*b*] because new information has been obtained for Great Britain, South Africa, and Japan. The contours of the geoid referred to this ellipsoid are plotted in Figure 10. Figure 11 illustrates the differences caused by the use of other reference ellipsoids. As in the case of the western hemisphere, the Hough ellipsoid is a good approximation to the best solution, while the International ellipsoid, even in its best fitting orientation, does not fit well.

The Krasovskiy ellipsoid, oriented as in the present Russian datum, was also investigated as a reference spheroid. As expected, it fits very well along the parallels of the northern hemisphere where it was derived. It does not fit in Africa, or in the Far East.

The Ice Age—In Figures 4 and 10 a remarkable correlation appears between the geoid contours and the areas of Pleistocene glaciation. The connection between them is discussed in a separate article [*Fischer,* 1959].

Triaxiality—The representation of the geoid on a best-fitting ellipsoid should solve the problem of triaxiality. In Figure 10 the highest and lowest areas beyond the ± 20 m contour line are marked. They seem to be arranged in a distinct undulation pattern. The longitudinal distance between the high and low zones, however, is not 90° but only 45°. This would suggest a system of two major axes perpendicular to each other, with two minor axes midway between them; in other words, a pentaxial rather than a triaxial spheroid. A continuation of this undulation pattern toward the west could be seen in the hump one might expect on the ice cap of Greenland and in the North Atlantic, the depression of the Hudson Bay and the hump again in the area of about 135° west longitude. Before assuming an undulating spheroid, triaxial or otherwise, for the whole earth, it would have to be seen whether the same pattern holds within the entire longitude zone. Apparently, this is not the case. Therefore, the ellipsoid of revolution still seems to be our best choice of reference ellipsoid.

The Columbus Geoid—Gravimetrically computed geoidal heights are deviations from a

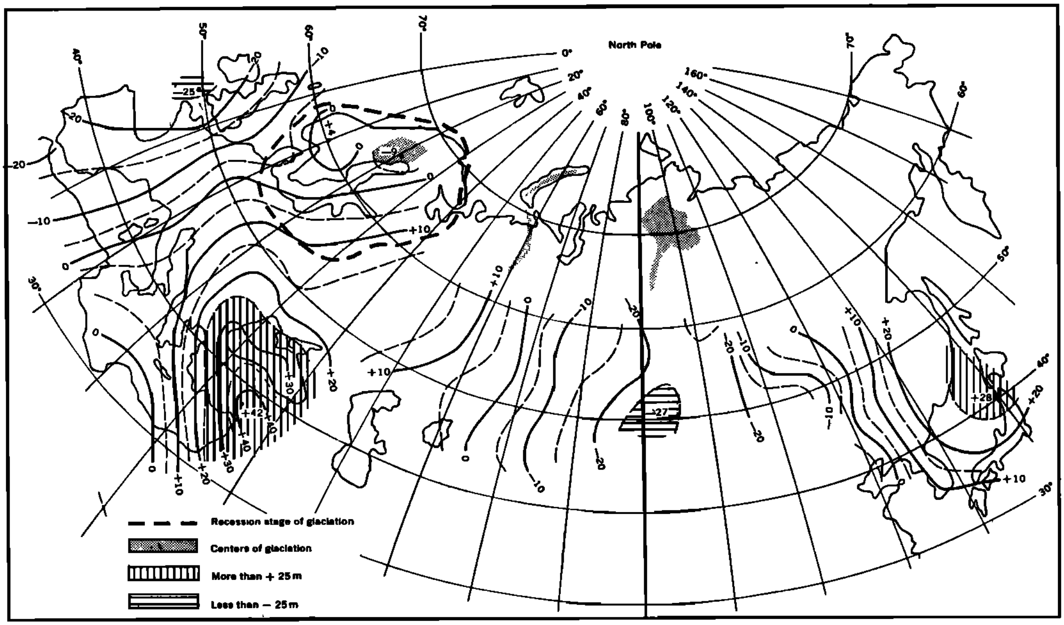

FIG. 10—Eastern hemisphere, geoid contours in meters, best fitting ellipsoid

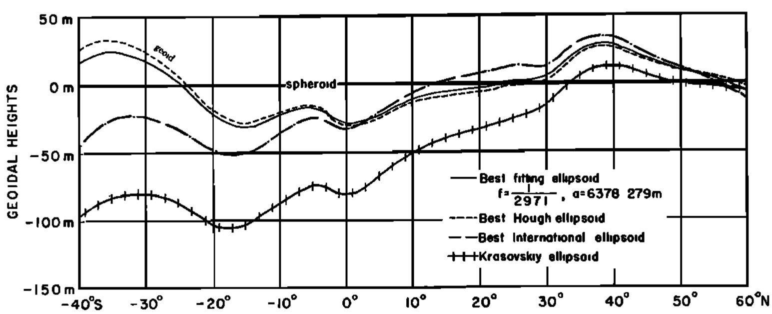

Fig. 11—Geoidal heights relative to various ellipsoids, Africa, 30th meridian

norm based on the adopted gravity formula. The Columbus Geoid [*Heiskanen*, 1957] is based on the International gravity formula in which the value of the flattening $f = 1/297$. The norm, therefore, is an ellipsoid of revolution (supposing the longitude term of that formula to be neglected) with a predetermined shape. The size of that ellipsoid or the distance of a particular point from the center of the earth cannot be determined by this method alone, if we are interested in an accuracy of the order of 50 m as is discussed in astro-geodetic studies. We might envisage the gravimetric geoid as a surface that, within reason, can be stretched to scale, without effect on the geoidal elevations; something like an elastic wrist watch band. Matching an astro-geodetic to a gravimetric geoid of the same area imposes a definite scale on the latter. If the size of the gravimetric ellipsoid (the norm) is known, a conversion formula can be derived between the gravimetric and astro-geodetic systems.

The Columbus Geoid could be matched at 75 out of 131 points to the astro-geodetic geoid on the North American Datum. The least-squares solution for an ellipsoid of flattening $f = 1/297$, compatible with the Columbus Geoid elevations at these 75 points, gives the size of that ellipsoid as $a = 6{,}378{,}240 \pm 21$ m. The comparable astro-geodetic ellipsoid, by which we mean the best fitting ellipsoid with the same flattening of 1/297 for the same 75 points, has a

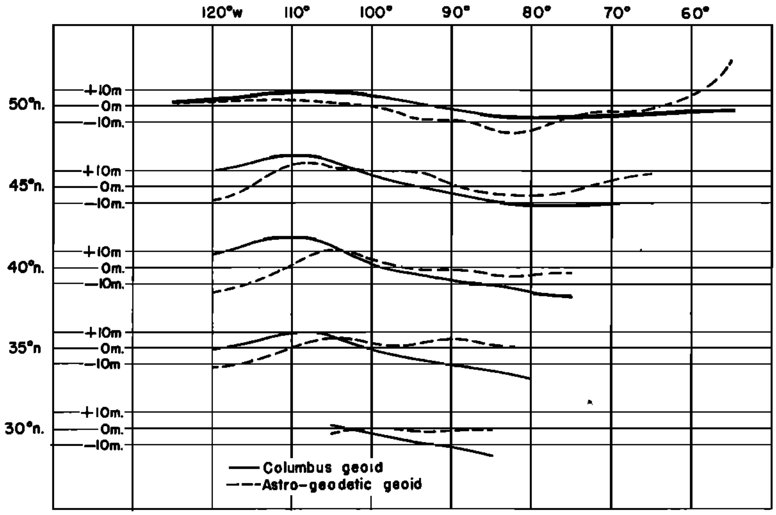

FIG. 12—Columbus Geoid in America, profiles along parallels

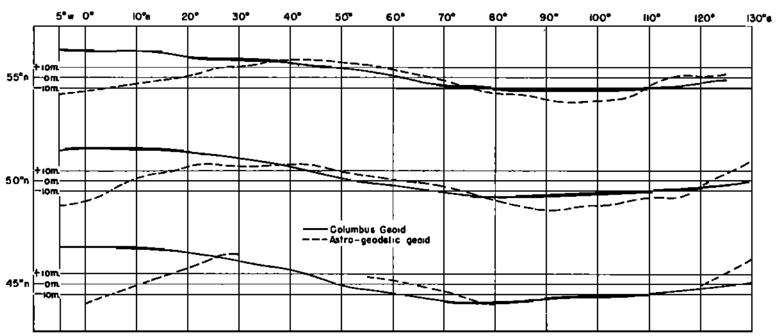

FIG. 13—Columbus Geoid in eastern hemisphere, profiles along parallels

size of $a = 6{,}378{,}248 \pm 25$ m, practically the same size as the above gravimetric ellipsoid. This may be taken as a criterion for good agreement. Figure 12 illustrates this agreement, comparing profiles of the Columbus Geoid with those of the astro-geodetic geoid, referred to this best fitting ellipsoid (not to the North American Datum). It can be seen that a certain tilt would make the profiles practically coincide, if we disregard certain areas where obviously more information is needed.

The Columbus Geoid of Europe, Africa, and Asia was matched at 127 points to the astro-geodetic geoid of the eastern hemisphere derived here. In the least-squares solution for the size of the gravimetric ellipsoid the semi-major axis a was found to be $6{,}378{,}214 \pm 8$ m. The comparable astro-geodetic ellipsoid has a size of $a = 6{,}378{,}270 \pm 10$ m; it does not differ greatly from the solution for all points, although the African points south of the equator and the northern points on the 65th parallel were omitted because they were not covered by the Columbus map. This time our criterion of good agreement has not been met, since the semi-major axes derived from the two geoid maps differ by more than 50 m. In Figure 13, the profiles along three parallels are compared. The astro-geodetic profiles of this Figure are again not referred to the European Datum but to the best fitting comparable ellipsoid. The Figure suggests that there is no single shift that could make both ends of the profiles coincide; the European parts could be made to fit at the expense of Asia; the Asiatic parts agree in a very generalized sense and that only at the expense of Europe. It is obvious that much more work is needed to find and remove the causes of disagreement. If the

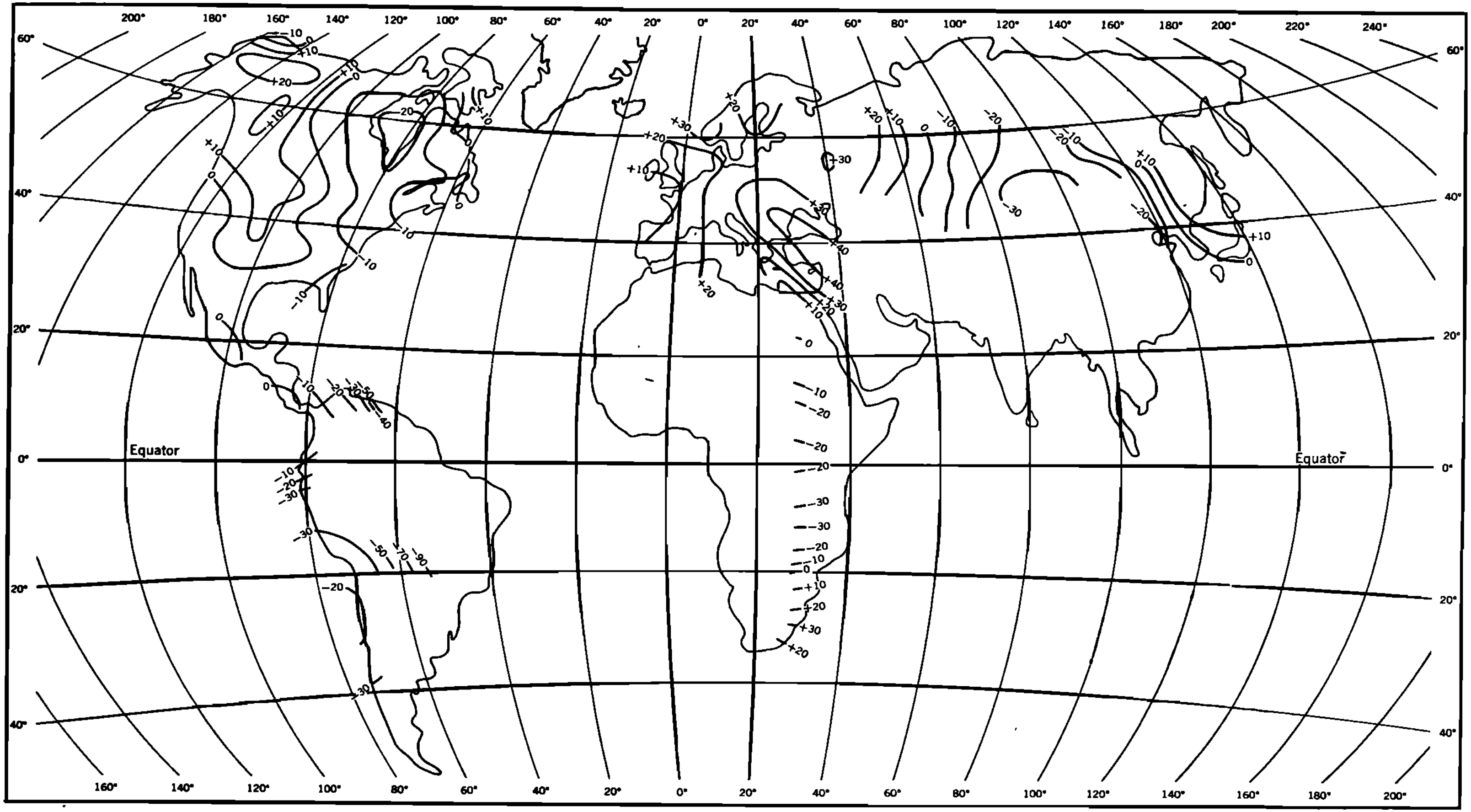

Fig. 14—Tentative World Datum, based on Hough ellipsoid and Columbus Geoid, geoid contours in meters

astro-geodetic geoid is at fault, it might be a reflection on the weakly documented section in the USSR. If the gravimetric geoid is the one that should be changed it might be for the reason of insufficient observations in certain areas.

A tentative World Datum—It is said that the gravimetric system is absolutely oriented, meaning that the center of the ellipsoid used as the norm is located at the center of the earth and its minor axis coincides with the axis of rotation. The theoretical relation to the center of the earth is the big advantage of the gravimetrical approach over the astro-geodetic approach, for it relates otherwise unconnected areas in a single system. On the other hand, the size of this ellipsoid cannot be determined without recourse to the astro-geodetic method. Thus the two methods should be combined and the combination used to connect the North American Datum with the European Datum and to devise a World Datum.

Matching the gravimetric geoid elevations to astro-geodetic geoid heights in order to determine the size of the corresponding gravimetric ellipsoid has yielded different sizes for different areas. For the western hemisphere we found a = 6,378,240 m, for the eastern hemisphere a = 6,378,214 m; for Europe alone *Lieberman* [1955] determined Tanni's ellipsoid as a = 6,378,160 m. This indicates that the gravimetric ellipsoid depends as much on the particular area as does the astro-geodetic ellipsoid. Our astro-geodetic studies have led to the conclusion that the Hough ellipsoid is our present best guess for the figure of the earth as a whole. We will, therefore, assume this size also for the gravimetric norm. A least-squares solution for the 75 points in the western and 127 points in the eastern hemisphere, minimizing the differences between astro-geodetic and gravimetric geoid heights, determines the best orientation of the North American and European systems in relation to the so-called gravimetric Hough ellipsoid and leads to conversion formulas between them.

The tentative World Datum is defined by the parameters of the Hough ellipsoid (a = 6,378,270 m, $f = 1/297$) and the World geodetic coordinates of Meades Ranch, or any other point chosen as origin. Figure 14 is a geoid contour map based on this World Datum. The elevations shown are essentially astro-geodetic elevations. Only the orientation was determined from gravimetric information and used to convert to the World Datum the elevations previously determined on the European or North American Datum respectively. While theoretically the astro-geodetic and gravimetric geoid elevations should agree, this World Geoid map and the Columbus Geoid map do not agree in detail at this time.

The World Datum defined here is as tentative as are the Hough ellipsoid and the Columbus geoid, on which it is based. It must be borne in mind that only a small fraction of the earth's surface is covered at present by astro-geodetic or gravimetric data. Future information will certainly modify our numerical results, especially information obtained from two projects: the North Atlantic tie between the hemispheres and the artificial satellite.

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